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Short communication

# New empirical expressions to [correlate](http://www.elsevier.com/locate/tca) [solubilities](http://www.elsevier.com/locate/tca) [o](http://www.elsevier.com/locate/tca)f solids in supercritical carbon dioxide

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#### ABSTRACT

Supercritical processes are gaining importance in the last few years in the food, environmental and pharmaceutical product processing. The design of any supercritical process needs accurate experimental data on solubilities of solids in the supercritical fluids (SCFs). The empirical equations are quite successful in correlating the solubilities of solid compounds in SCF both in the presence and absence of cosolvents. In this work, existing solvate complex models are discussed and a new set of empirical equations is proposed. These equations correlate the solubilities of solids in supercritical carbon dioxide (both in the presence and absence of cosolvents) as a function of temperature, density of supercritical carbon dioxide and the mole fraction of cosolvent. The accuracy of the proposed models was evaluated by correlating 15 binary and 18 ternary systems. The proposed models provided the best overall correlations.

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#### **1. Introduction**

Supercritical fluid technology (SFT) finds applications in chemical, biochemical, pharmaceutical and food processing industries. Supercritical fluids (SCFs) have diffusivities between that of gases and liquids, compressibilities comparable to gases, densities comparable to liquids and negligible surface tension. These properties make them attractive solvents for many industrial applications [1]. Carbon dioxide  $(CO_2)$  is commonly used as a supercritical fluid because it is nontoxic and nonflammable. Polar substances are poorly soluble in  $SCCO<sub>2</sub>$  because of its lack of polarity. The solubilities can be enhanced by adding cosolvents [2,3]. Solubility data in SCFs are important for the successful impl[eme](#page-4-0)ntation of SFT. An exhaustive account of experimental data on the solubilities of various organics in SCFs is available [4]. The experimental determination of solubilities of organic solids in SCFs at various temperatures and pressures is expensiv[e.](#page-4-0) [The](#page-4-0)refore, modeling and prediction of solubilities is essential [5].

Models used for correlating solubilities can be broadly classified as equation of state (EOS) ba[sed](#page-4-0) [m](#page-4-0)odels and empirical models. EOS based models require parameters such as critical constants and the sublimation pressure of the solids. These are normally not available for many high [molec](#page-4-0)ular weight compounds and are calculated using group contribution methods, which could lead to error. Therefore, empirical models are used in correlating solubilities of solids in  $SCCO<sub>2</sub>$  [6-10].

In this study, solvate complex models published in the literature are discussed. A new set of empirical equations is proposed to correlate solubilities of high molecular weight solids in  $SCCO<sub>2</sub>$ . The accuracy of the proposed models was evaluated by correlating solubilities of [various](#page-4-0) [s](#page-4-0)olids in supercritical fluids both in the presence and absence of cosolvents for several systems.

#### **2. Theoretical section**

#### 2.1. Solubilities of organic solids in SCF

Chrastil [11] correlated the solubility of a solute  $(y_2)$  in SCFs to the density (  $\rho_1$  , g mL $^{-1}$  ) and temperature (T, K) as

$$
\ln(y_2) = A_0 \ln(\rho_1) + \frac{A_1}{T} + A_2 \tag{1}
$$

where  $A_0$ - $A_2$  are the model constants that can be estimated from experimental solubility data in SCF.

The solubility of a solute in SCFs can be correlated to the density and temperature by the modified Adachi-Lu equation [12]

$$
\ln(y_2) = (B_0 + B_1 \rho_1) \ln(\rho_1) + \frac{B_2}{T} + B_3 \tag{2}
$$

where  $B_0 - B_3$  are the model constants.

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<span id="page-1-0"></span>**Table 1a** Range and source of binary system solubility data of various solutes in SCCO<sub>2</sub>.

<b>SN</b>	Solute	T(K)	P(MPa)	Mole fraction ( $\times$ 10 <sup>5</sup> )	$N_i$	Reference
	2-Chlorobiphenyl	$313 - 333$	$10.0 - 30.0$	$40.1 - 561.0$	48	$[18]$
2	4-Chlorobiphenyl	313-333	$10.0 - 30.0$	$47.0 - 1740.0$	48	[18]
3	2,2'-Dichlorobiphenyl	313-333	$10.0 - 30.0$	27.2-1210.0	48	[18]
4	4,4'-Dichlorobiphenyl	313-333	$10.0 - 30.0$	14.3-175.0	48	$[18]$
5	2.4'.5-Trichlorobiphenyl	313-333	$10.0 - 30.0$	31.4-393.0	48	$[18]$
6	3,3',4,4'-Tetrachlorobiphenyl	313-333	$10.0 - 30.0$	$0.08 - 9.71$	48	$[18]$
	2,2',4,5,5'-Pentachlorobiphenyl	313-333	$10.0 - 30.0$	$2.3 - 157.0$	48	$[18]$
8	2,2',4,4',5,5'-Hexachlorobiphenyl	$313 - 333$	$10.0 - 30.0$	$1.5 - 67.7$	48	[18]
9	2,2',3,4,4',5,5'-Heptachlorobiphenyl	313-333	$10.0 - 30.0$	$0.8 - 52.4$	48	[18]
10	Benzoic acid	$308 - 343$	$11.0 - 36.4$	22.0-1280.0	39	$[19]$
11	Anthracene	313-333	11.8-35.59	$2.2 - 18.3$	90	[20]
12	Pyrene	313-333	$11.4 - 35.0$	$2.2 - 55.0$	96	[20]
13	Perylene	323-333	$17.2 - 35.0$	$0.062 - 0.22$	19	[20]
14	Flurbiprofen	$303 - 323$	$8.9 - 24.5$	1.672-19.63	27	[21]
15	Aspirin	308-328	$12.0 - 25.0$	$6.3 - 34.7$	24	$[22]$

González et al. [13] extended the Chrastil equation to model the solubilities in presence of a cosolvent as

$$
\ln(y_2') = C_0 \ln(\rho_1) + C_1 \ln(y_3) + \frac{C_2}{T} + C_3 \tag{3}
$$

where  $y_2'$  [is s](#page-4-0)olubility in presence of cosolvents and  $y_3$  is the mole fraction of cosolvent and  $C_0$ – $C_3$  are model constants.

The solubility of solute in SCFs can be correlated to the density and temperature as [14,15]

$$
\ln(y_2) = E_0 \ln(\rho_1 T) + E_1 \rho_1 + \frac{E_2}{T} + E_3 \tag{4}
$$

where  $E_0 - E_3$  are the model constants.

Li et al. [\[14\]](#page-4-0) correlated the solubility of a solute in SCFs in presence of cosolvent as

$$
\ln(y_2') = F_0 \ln(y_3 \rho_1 T) + F_1 \rho_1 + \frac{F_2}{T} + F_3 \tag{5}
$$

[wher](#page-4-0)e  $F_0-F_3$  are the model constants.

Garlapati and Madras [16] correlated the solubility of a solute in SCFs to the density and temperature as

$$
\ln(y_2) = G_0 \ln(\rho_1 T) + \frac{G_1}{T} + G_2 \tag{6}
$$

where  $G_0-G_2$  [are the](#page-4-0) model constants

Garlapati and Madras [16] correlated the solubilities of a solute in SCFs in presence of cosolvents as

$$
\ln(y_2') = H_0 \ln(\rho_1 T) + H_1 \ln(y_3 T) + H_2 \ln(T) + \frac{H_3}{T} + H_4 \tag{7}
$$

where  $H_0$ - $H_4$  [are th](#page-4-0)e model constants.

The density-based models (Eqs.  $(1)-(7)$ ) i[ndica](#page-4-0)te the existence of:

- a. non-linear relationship between ln( $y_2$ ) and density ( $\rho_1$ ) when temperature is consta[nt \(Isoth](#page-0-0)ermal condition).
- b. non-linear relationship between  $ln(y_2)$  and temperature (T) when density is constant (Isopycnic condition).
- c. linear relation between ln(y<sub>2</sub>) and ln( $\rho_{1T}$ ) in a certain range of density and temperature.
- d. non-linear relationship between  $\ln(y'_2)$  and density  $(\rho_1)$  when cosolvent mole fraction and temperature are constant.
- e. non-linear relationship between  $\ln(y'_2)$  and temperature  $(T)$ when cosolvent mole fraction and density are constant.
- f. linear relation between ln( $y_2^\prime$ ) and ln( $y_3$ ) when temperature and density are constant.
- g. linear relation between  $\ln(y'_2)$  and  $\ln(y_3\rho_{1T})$  in a certain range of density, temperature and cosolvent mole fraction.

Taking into account all the above observations for a better correlation, the following empirical equation was proposed for the solubilities of organic solids in  $SCCO<sub>2</sub>$  by Jouyban et al. [8,9]:

$$
\ln(y_2) = m_0 + m_1 \rho_1 + m_2 P^2 + m_3 PT + \frac{m_4 T}{P} + m_5 \ln(\rho_1)
$$
 (8)

where  $m_0$ - $m_5$  are model constants. We pro[pose a](#page-4-0) new model as

$$
\ln(y_2) = M_0 + (M_1 + M_2 \rho_1) \ln(\rho_1) + \frac{M_3}{T} + M_4 \ln(\rho_1 T) \tag{9}
$$

#### **Table 1b**

Range and source of ternary system solubility data of various solutes in presence of different cosolvents in SCCO<sub>2</sub>.



<span id="page-2-0"></span>**Table 2a** Correlation results based on the proposed model (Eq. (9)) for various systems.

SN <sup>a</sup>	$M_0$	$M_1$	$M_2 \ (\times 10^3)$	$M_3$	$M_4$	AARD %	$R_{\text{adj}}$	<i>F</i> value
	$-54.59$	$-4.56$	0.30	$-572$	6.37	3.74	0.9962	5030.25
	$-80.37$	$-9.12$	0.53	$-1492$	11.10	4.22	0.9970	155.59
	$-77.20$	$-9.33$	0.59	$-2313$	11.10	2.79	0.9987	20350.30
4	$-78.50$	$-9.10$	0.20	$-2311$	11.10	4.01	0.9940	9796.58
5	$-82.12$	$-9.48$	0.31	$-196$	11.10	3.65	0.9951	6297.23
6	$-77.57$	$-9.19$	0.83	$-4558$	11.10	5.03	0.9975	10374.77
	$-82.20$	$-9.53$	0.78	$-1259$	11.10	4.02	0.9982	6917.29
8	$-83.75$	$-9.16$	0.50	$-1332$	11.10	4.36	0.9966	6249.93
9	$-75.93$	$-9.64$	0.77	$-3460$	11.10	3.44	0.9983	15546.6
10	$-29.51$	$-1.10$	0.43	$-4794$	3.51	0.30	0.9898	1994.91
11	101.99	24.18	$-0.20$	$-12070$	$-18.82$	6.03	0.9827	3201.78
12	$-99.99$	$-11.09$	0.491	$-678$	13.27	4.81	0.9936	5062.29
13	$-48.47$	$-6.71$	0.79	$-575$	6.20	1.16	0.9984	4980.43
14	21.25	$-2.01$	1.38	$-8401$	0.18	6.07	0.9848	1301.16
15	$-5.62$	1.99	0.36	$-6160$	0.09	5.41	0.9744	976.84

<sup>a</sup> SN, the system name and number is same as in Table 1a.

#### **Table 2b**

Correlation results based on the Jouyban et al. model (Eq. (8)) for various systems.



<sup>a</sup> SN, the system name and number same as Table 1a.

where M<sub>0</sub>–M<sub>4</sub> are the model constants and  $\rho$  is the density of the SCCO<sub>2</sub> at different pressures and temperatures.

where  $n_0 - n_6$  are model constants. We propose a new model as

For the solubilities of [organic](#page-1-0) solids in  $SCCO<sub>2</sub>$  in presence of cosolvent, Jouyban et al. [10] proposed:

$$
\ln(y_2') = N_0 + (N_1)\ln(\rho_1) + N_2\rho_1 + \frac{N_3}{T} + N_4\ln(T) + N_5\ln(y_3) + N_6\ln(y_3\rho_1T)
$$
 (11)

$$
\ln(y_2') = n_0 + n_1 y_3 + n_2 \rho_1 + n_3 P^2 + n_4 P T + \frac{n_5 T}{P} + n_6 \ln(\rho_1)
$$
 (10)

where  $N_0 - N_6$  are the model constants.

**Table 3a** Correlation results based on the proposed model (Eq. (11)) for various systems.

SN <sup>a</sup>	$N_0$	$N_1$	$N_2$ ( $\times$ 10 <sup>3</sup> )	$N_3$	$N_4$	N <sub>5</sub>	$N_6$	AARD %	$R_{\text{adj}}$	F value
	$-0.99$	0.243	4.88	$-3055$	0.070	$-0.66$	$-0.325$	5.38	0.9896	2143.99
2	$-7.29$	0.736	4.40	$-4155$	1.098	1.467	0.5319	4.67	0.9957	4686.58
3	$-7.67$	1.159	3.60	$-5120$	1.001	1.595	0.715	3.51	0.9973	18971.23
4	$-5.18$	1.887	14.65	$-5551$	$-0.080$	0.764	0.574	3.71	0.9944	9834.88
5	$-10.39$	1.093	1.30	$-3099$	1.001	2.504	0.776	3.53	0.9939	8016.04
6	$-7.29$	1.156	5.79	$-7534$	1.001	1.467	0.700	4.80	0.9974	14155.31
	$-8.71$	0.775	5.51	$-4306$	1.001	1.942	0.562	3.62	0.9981	11761.56
8	$-9.46$	0.877	3.86	$-4180$	1.001	2.192	0.638	5.36	0.9943	3625.70
9	2.51	1.507	5.48	$-7392$	$-1.001$	$-1.807$	$-0.230$	3.93	0.997	15326.54
10	$-10.33$	0.729	2.35	$-2232$	1.098	2.482	0.632	3.36	0.9955	5379.77
11	$-6.04$	0.543	4.90	$-3966$	1.001	1.051	0.359	3.83	0.9973	4945.54
12	$-7.77$	1.10	3.74	$-4875$	1.001	1.638	0.690	3.55	0.9974	15768.46
13	$-10.12$	1.51	0.02	$-4741$	1.001	2.413	0.973	3.97	0.993	8446.11
14	$-10.60$	1.03	1.46	$-2780$	1.001	2.572	0.752	3.52	0.9941	7916.72
15	$-6.79$	0.956	6.22	$-7153$	1.001	1.302	0.585	5.17	0.9971	8310.66
16	$-9.30$	0.813	5.22	$-4023$	1.001	2.138	0.600	4.33	0.9976	8869.72
17	$-10.89$	1.278	2.98	$-3618$	0.694	2.671	0.767	5.80	0.9946	2863.92
18	$-6.01$	0.642	5.91	$-5777$	1.001	1.041	0.403	5.70	0.9961	6148.30

<sup>a</sup> SN, the system name and number same as Table 1b.





a SN, the system name and number same as Table 1b.

#### **Table 4**

Statistical summary of the models proposed for binary (Tables 2a and 2b) and ternary (Tables 3a and 3b) systems.



<sup>a</sup> AARD calculated using Eq. [\(12\)](#page-2-0).

 $\delta$   $R_{\text{adj}} = \sqrt{\vert R^{2}-(Q(1-R^{2})/(N-Q-1)) \vert}$  where R is correlation coefficient, N is the number of experimental data points in each set, Q is number of independent variables in each equation.

 $c$  F value is the ratio of mean square regression to the mean square residual.

 $d$  The binary systems mean AARDs are significantly different from 4.52% (paired t-test,  $P < 0.005$ ).

<sup>e</sup> The binary systems mean  $R_{\text{adj}}$  are significantly different from 0.993 (paired t-test, P < 0.005).

f The binary systems mean F values are significantly different from 6549.06 (paired t-test,  $P < 0.005$ ).

<sup>g</sup> The binary systems mean AARDs are significantly different from 4.3% (paired t-test, P<0.0005).

<sup>h</sup> The binary systems mean  $R_{\text{adj}}$  are significantly different from 0.9955 (paired t-test, P < 0.0005).

<sup>i</sup> The binary systems mean F values are significantly different from 8731.72 (paired t-test, P<0.0005)

#### 2.2. Methodology

The density of supercritical carbon dioxide required for themodeling was determined with Span and Wagner equation of state [17]. The optimization procedure reduces the percentage average absolute deviation, AARD % defined as

$$
AARD \ \mathcal{Z} = \frac{100}{N_i - Z} \sum_{i=1}^{N_i} \frac{\left| y_2^{calc} - y_2^{exp} \right|}{y_2^{exp}}
$$
 (12)

where  $N_i$  is the number of data points, Z is the curve fitting parameters for each model,  $y_2$  represents the molar solubility of the solute. The superscripts calc and exp denotes the calculated and experimental values, respectively. The Z values for Eqs. (8), (9), (10) and (11) are 6, 5, 7 and 7, respectively. The correlation of the experimental solubility data requires an optimization process where the constants for empirical model[s were](#page-1-0) determined by using the non-linear regression. Tables 1a and 1b show the sy[stems](#page-2-0) studied both in the presence and absence of [cos](#page-1-0)[olven](#page-2-0)ts.

#### **3. Results**

To illustrate the usefulness of the current models, the solubilities of solids in  $SCCO<sub>2</sub>$  and  $SCCO<sub>2</sub>$  along with cosolvent were compared with the existing solvate complex models, i.e., Eqs. (1)–(9) (see the supplementary information). Clearly, the equation proposed by Jouyban et al. (Eq. (8)) is superior to the existing models. However, the model proposed in this study (Eq. (9)) is even superior to this model, as shown in Tables 2a and 2b and Tables 3a and 3b.

From Tables 2a and 2b, it is clear that th[e](#page-0-0) [propos](#page-0-0)[ed](#page-4-0) [mo](#page-4-0)dels cor[relate](#page-4-0) [the](#page-4-0) [solubility](#page-4-0) [b](#page-4-0)etter. Tables 3a and 3b show that the newly propos[ed](#page-1-0) [m](#page-1-0)odel correlates the solubilities of solids in supercritical fluid in presence of coso[lvent](#page-1-0)s better than existing models. The propose[d](#page-2-0) [models,](#page-2-0) [Eqs.](#page-2-0) (9) and [\(11\),](#page-2-0) [and](#page-2-0) [the](#page-2-0) [prev](#page-2-0)ious models, Eqs. [\(8\)](#page-2-0) [and](#page-2-0) [\(10\),](#page-2-0) [are](#page-2-0) statistically significantly (paired  $t$ -test,  $P < 0.005$ ) different. The ad[justed](#page-2-0) [correlation](#page-2-0) coefficient,  $R_{\text{adj}}$  and F values for binary and ternary systems are reported in Table 4.

# **[4.](#page-1-0) Conclusions**

New empirical models were developed for correlating the solubilities of solids in supercritical fluids with and without cosolvents. <span id="page-4-0"></span>The new models proposed in this work were successful in correlating solubility of solids in  $SCCO<sub>2</sub>$  with and without cosolvent and were found to correlate the systems better than existing models.

## **Acknowledgments**

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#### **Appendix A. Supplementary data**

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.tca.2009.12.004.

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